Asset Price Bubbles

with and without rational expectations

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Personal note

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Basic Setup

- a share entitles its owner to a stream of dividends $\{d_t\}$ governed by a Markov chain defined on a state space $S \in \{1, 2\}$.
- Dividend obeys

$$d_t = \begin{cases} 0 & \text{if } s_t = 1; \\ 1 & \text{if } s_t = 2. \end{cases}$$

Plan of talk

- Use two versions of a basic setup to describe two theories of the price of the asset
  - Rational expectations (communism of beliefs)
  - Not rational expectations (diversity of beliefs)
- In both versions there are bubbles

Homogeneous Beliefs

- States evolve according to a a Markov transition matrix P with typical element $P(i,j) = \mathrm{Prob}(s_{t+1} = j | s_t = i)$.
- The transition matrix is shared by nature and agents (rational expectations)

Shiller's Equation

- $p_t = \beta E_t[d_{t+1} + p_{t+1}]$
- $p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t+j}$
- $\begin{bmatrix} p(0) \\ p(1) \end{bmatrix} = \sum_{j=1}^{\infty} \beta^j P^j \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \beta[I - \beta P]^{-1} P \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Blanchard-Watson RE bubble

- $p_t = \beta E_t[p_{t+1}]$ is satisfied by

$$p_t = m_t \beta^{-t}$$

  for *any* martingale $E_t m_{t+1} = m_t$.
- A rational expectations bubble.

General solution

- $$p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t+j} + m_t \beta^{-t}$$
- Fundamental value plus bubble
- Both terms rely heavily on rational expectations

Examples

- $m_{t+1} = a_{t+1} m_t$ where

$$a_{t+1} = \exp(-\sigma\varepsilon_{t+1} - \frac{\sigma^2}{2}),\ \varepsilon_{t+1} \sim \mathcal{N}(0,1)$$

- $$m_{t+1} = \begin{cases} \phi m_t \text{ with probability } \frac{1}{\phi} \\ 0 \text{ with probabilty } 1 - \frac{1}{\phi} \end{cases}$$

  where $\phi > 1$.

Econometric Testing

- Tests use rational expectations econometrics
- Imrohoroglu, P.C.B. Phillips and coauthors

Heterogeneous Beliefs

Harrison-Kreps (1978), Scheinkman (2014)
Two types $h = a, b$ of investors are distinguished only by their beliefs about a Markov transition matrix P with typical element $P(i,j) = \mathrm{Prob}(s_{t+1} = j | s_t = i)$.

Heterogeneous Beliefs

- Agents of type a believe the transition matrix

$$P_a = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

- Associated with transition matrix P_a is the invariant distribution $\pi_A = \begin{bmatrix} .57 & .43 \end{bmatrix}$
- Agents of type b think the transition matrix is

$$P_b = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

- Associated with transition matrix P_b is the invariant distribution $\pi_B = \begin{bmatrix} .43 & .57 \end{bmatrix}$.

Temporary Optimism and Pessimism

- Stochastically alternating temporary optimism and pessimism.
- State 2 is the high dividend state.
- In state 1, a type *a* agent is more optimistic about next period's dividend than is a type *b* agent
- In state 2, a type *b* agent is more optimistic about next period's dividend.
- The invariant distributions π_A and π_B tell us that a type B person is more optimistic about the dividend process in the long run than is a type A person.

- An owner of the asset at the end of time t is entitled to the dividend at time $t + 1$ and the right to sell the asset at time $t + 1$.
- Both types of investors are risk-neutral and both have the same fixed discount factor $\beta \in (0, 1)$.

Key assumptions

- Assumption about the number of shares outstanding relative to the resources that our two types of investors can invest in the stock.
- Assume that both types of investor have access to enough resources (either wealth or the capacity to borrow) to purchase the entire stock of the asset
- Short sales are prohibited
- Incomplete markets

Equilbrium price function

Agents know a function mapping the state s_t into the equilibrium price $p(s_t)$. When they choose whether to purchase or sell the asset at t, agents know s_t.

Equilibrium price functions

- Assumption 1: There is only one type of agent, either *a* or *b*.
- Assumption 2: There are two types of agent differentiated only by their beliefs. Each type of agent has sufficient resources to purchase all of the asset.

Single Belief Prices

- Suppose that there is only one type of investor, either of type *a* or *b*, and that this investor always "prices the asset"
- Let $p_h = \begin{bmatrix} p_h(1) \\ p_h(2) \end{bmatrix}$ be the equilibrium price vector when all investors are of type h

- The price today equals the expected discounted value of tomorrow's dividend and tomorrow's price of the asset
- $$p_h(s) = \beta \left(P_h(s,1) p_h(1) + P_h(s,2)(1 + p_h(2)) \right), \quad s = 1, 2$$
- $$\begin{bmatrix} p_h(1) \\ p_h(2) \end{bmatrix} = \beta [I - \beta P_h]^{-1} P_h \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Pricing under Heterogeneous Beliefs

- The marginal investor who prices the asset in state s is of type a if

$$P_a(s,1)\bar{p}(1)+P_a(s,2)(1+\bar{p}(2)) > P_b(s,1)\bar{p}(1)+P_b(s,2)(1+\bar{p}(2))$$

- The marginal investor is of type b if

$$P_a(s,1)\bar{p}(1)+P_a(s,2)(1+\bar{p}(2)) < P_b(s,1)\bar{p}(1)+P_b(s,2)(1+\bar{p}(2))$$

- Thus the marginal investor is the (temporarily) optimistic type.

Harrison-Kreps Functional Equation

$$
\begin{aligned}
\bar{p}(s) &= \beta \max\{A, B\} \\
A &= P_a(s,1)\bar{p}(1) + P_a(s,2)(1+\bar{p}(2)) \\
B &= P_b(s,1)\bar{p}(1) + P_b(s,2)(1+\bar{p}(2))
\end{aligned}
$$

for $s = 1, 2$

Individual Values

- Investors of type a are willing to pay the following price for the asset

$$\hat{p}_a(s) = \begin{cases} \bar{p}(1) & \text{if } s_t = 1 \\ \beta(P_a(2,1)\bar{p}(1) + P_a(2,2)(1 + \bar{p}(2))) & \text{if } s_t = 2 \end{cases}$$

- Investors of type b are willing to pay the following price for the asset

$$\hat{p}_b(s) = \begin{cases} \beta(P_b(1,1)\bar{p}(1) + P_b(1,2)(1 + \bar{p}(2))) & \text{if } s_t = 1 \\ \bar{p}(2) & \text{if } s_t = 2 \end{cases}$$

Prices

s_t	1	2
p_a	1.33	1.22
p_b	1.45	1.91
$\overline{p}$	1.85	2.08
$\hat{p}_a$	1.85	1.69
$\hat{p}_b$	1.69	2.08

Row 1: equilibrium price function p_a under homogeneous beliefs P_a.
Row 2: equilibrium price function p_b under homogeneous beliefs P_b.
Row 3: equilibrium price function under heterogeneous beliefs with optimistic marginal investors.
Row 4: type a agents are willing to pay $\hat{p}_a$ for asset.
Row 5: type b agents are willing to pay $\hat{p}_b$ for the asset. $\beta = .75$.

Bubble

The third row of the table reports equilibrium prices that solve Harrison and Kreps's functional equation when $\beta = .75$.

- The type that is (temporarily) optimistic about s_{t+1} prices the asset in state s_t.
- Equilibrium prices $\overline{p}$ in the heterogeneous beliefs economy exceed what every prospective investor regards as the fundamental value of the asset in each possible state.
- Each purchaser of the asset pays more than he believes its future dividends are worth because he expects to have the option to sell the asset later to another investor who will value the asset more highly than he will.

Applications, I

- Compared to a homogeneous beliefs setting, high volume occurs when the Harrison-Kreps pricing formula prevails.
- Type *a* agents sell the entire stock of the asset to type *b* agents every time the state switches from $s_t = 1$ to $s_t = 2$.
- Type *b* agents sell the asset to type *a* agents every time the state switches from $s_t = 2$ to $s_t = 1$.
- Scheinkman (2014) takes this as a strength of the model because he observes high volume during "famous bubbles".

Applications, II

- If the *supply* of the asset can be increased sufficiently either physically (more “houses” are built) or artificially (ways are invented to short sell “houses”), bubbles end when the supply of the asset has grown enough to outstrip optimistic investors' resources for purchasing the asset.

Applications, III

- Scheinkman (2014) extracts insights about effects of financial regulations on bubbles. He emphasizes how prohibiting short sales and limiting leverage have opposite effects.

Other frameworks

Good bubbles with rational expectations:

- Money and government debt in overlapping generations models (Samuelson (1958)
- Relaxing collateral constraints in financial accelerator models (Bernanke-Gertler)